# Isotropic transverse *XY* chain with energy and magnetization currents

T. Antal,[1] Z. Rácz,[1,2] A. Rákos,[1] and G. M. Schütz[3]

[1]*Institute for Theoretical Physics, Eötvös University, 1088 Budapest, Puskin utca 5-7, Hungary*
[2]*Department of Physics, Theoretical Physics, University of Oxford, 1 Keble Road, Oxford, OX1 3NP, United Kingdom*
[3]*Institut für Festkörperforschung, Forschungszentrum Jülich, 52425 Jülich, Germany*

The ground-state correlations are investigated for an isotropic transverse *XY* chain that is constrained to carry either a current of magnetization ($J^M$) or a current of energy ($J^E$). We find that the effect of $J^M \neq 0$ on the large-distance decay of correlations is twofold: (i) Oscillations are introduced and (ii) the amplitude of the power-law decay increases with increasing current. The effect of energy current is more complex. Generically, correlations in current carrying states are found to decay faster than in the $J^E = 0$ states, contrary to expectations that correlations are increased by the presence of currents. However, increasing the current, one reaches a special line where the correlations become comparable to those of the $J^E = 0$ states. On this line, the symmetry of the ground state is enhanced and the transverse magnetization vanishes. A further increase of the current destroys the extra symmetry, but the transverse magnetization remains at the high-symmetry zero value. [S1063-651X(98)13405-0]

## I. INTRODUCTION

A general feature of nonequilibrium steady states is the presence of currents (fluxes) of some physical quantities such as energy, momentum, and charge. Thus the study of nonequilibrium steady states is, in some sense, a study of the effects of currents imposed on the system either by boundary conditions and driving fields or by competing dynamical processes. An interesting and much investigated effect of currents is the rather dramatic change in correlations. Namely, short-range correlations appear to change into long-range, power-law correlations as the currents are switched on [1,2]. This is not entirely surprising in the case of a global current since some conserved quantity is carried fast compared to diffusion and, in the absence of detailed balance, this may generate long-range effective interactions and, as a consequence, long-range correlations may appear [3]. It is clear, however, that the general picture should be more complicated since large currents often lead to chaotic behavior, which in turn results in weakened correlations [4]. Thus, we believe, the understanding of the interplay of currents and correlations in nonequilibrium steady states is a rather interesting and important task.

In order to achieve progress one tries to investigate simple models and, indeed, a large number of classical statistical models have been introduced for the study of nonequilibrium steady states [5]. Unfortunately, far from equilibrium, classical systems are not constrained by conditions such as detailed balance and there is much arbitrariness in defining the dynamics. In order to avoid such arbitrariness, we have started to investigate quantum systems [6] where the time evolution is defined without ambiguity by the usual rules of quantum mechanics.

Nonequilibrium steady states in a quantum system may be investigated by imposing a current on the system and studying the properties of the ground state thus generated. As an example, we studied the transverse Ising model [6] and found that, in the presence of an energy current, the exponentially decaying two-spin correlations changed into a power-law form, thus supporting the notion that switching on currents increases correlations. In view of the lack of detailed knowledge of the interplay of currents and correlations, here we probe the generality of the above result on the example of the isotropic transverse *XY* chain. In this system, we have a global conservation not only for the energy, as in case of the transverse Ising model, but also for the transverse magnetization and, consequently, the effect of both the energy and magnetization currents can be investigated. The *XY* chain is also interesting because it has power-law correlations already in the equilibrium state (i.e., in the state without any current) and so one might expect the system to be more sensitive to the introduction of currents. Indeed, in the case of the energy current we find a rather complex behavior (including an increase of the ground-state symmetry at special values of the current) and our findings for the steady-state correlations are at variance with those for the transverse Ising model. On the other hand, the changes we observe in the correlations due to a magnetization current allow for a straightforward interpretation in terms of increasing correlations due to presence of a current.

The effects of magnetization current are treated exactly (Sec. II), while the correlations in the presence of an energy current are calculated by a combination of analytical and exact numerical methods in Sec. III. A summary and final comments are contained in Sec. IV.

## II. MAGNETIZATION CURRENT

Our starting point is the Hamiltonian of the $d = 1$ isotropic *XY* model in a transverse field $h$:

$$H^{XY} = -\sum_{\ell=1}^{N} (s_\ell^x s_{\ell+1}^x + s_\ell^y s_{\ell+1}^y + h s_\ell^z), \quad (1)$$

where the spins are represented by Pauli spin matrices $s_\ell^\alpha$ ($\alpha = x, y, z$) at sites $\ell = 1, 2, \ldots, N$ of a one-dimensional peri-

odic chain ($s_{N+1}^{\alpha} = s_1^{\alpha}$). The transverse field $h$ is measured in units of the Ising coupling $J$, which is set to $J = 1$ throughout this paper. This model can be solved exactly [7] since it can be transformed by a Jordan-Wigner transformation into a set of free fermions with wave number $k$ and of energy

$$\Lambda_k = -\cos k - h. \tag{2}$$

In this model, not only the total energy but also the $z$ component of the total magnetization $M^z = \Sigma_{\ell} s_{\ell}^z$ is conserved. As a result, one can write down a continuity equation for the local magnetization $s_{\ell}^z$:

$$\dot{s}_{\ell}^z = i[H^{XY}, s_{\ell}^z] = j_{\ell-1}^M - j_{\ell}^M \tag{3}$$

and this defines the magnetization current through the bonds

$$j_{\ell}^M = s_{\ell}^y s_{\ell+1}^x - s_{\ell}^x s_{\ell+1}^y. \tag{4}$$

A macroscopic current can now be defined as

$$J^M = \sum_{\ell=1}^{N} j_{\ell}^M \tag{5}$$

and one can recognize $J^M$ as the Hamiltonian of the Dzyaloshinskii-Moriya interaction [8]. Somewhat surprisingly, the same expression emerged as the energy current in the case of the transverse Ising model [6].

Our aim is to find the lowest-energy state among the states carrying a given current. Since $[H^{XY}, J^M] = 0$, the problem can be solved using the Lagrange multiplier method, i.e., we diagonalize the Hamiltonian

$$H^M = H^{XY} - \lambda J^M, \tag{6}$$

where $\lambda$ is a Lagrange multiplier. The ground state of $H^M$ can be considered as a current-carrying steady state of $H^{XY}$ at zero temperature. Note that, without loss of generality, we can assume $h \geq 0$ and $\lambda \geq 0$.

The Hamiltonian $H^M$ is diagonalized using the same transformations that diagonalize $H^{XY}$ and we get the following spectrum in the thermodynamic limit:

$$\Lambda_k = \frac{1}{\cos\varphi}[-\cos(k-\varphi) - \tilde{h}], \tag{7}$$

where $\varphi = \arctan\lambda$ and an effective field $\tilde{h} = h\cos\varphi$ has been introduced. One can see that the spectrum is similar to that of $H^{XY}$ with the wave number shifted by $\varphi$. It should be mentioned here that the above result is not new. It is implicit in the Bethe-ansatz solution of the anisotropic Heisenberg chain with twisted toroidal boundary conditions [9] and appears in various forms in studies of the Dzyaloshinskii-Moriya interaction [10] and of the associated Berry phase [11].

It is remarkable that $H^M$ can be transformed into $H^{XY}$ [Eq. (1)] using a unitary transformation

$$Q = \exp i \sum_{\ell=1}^{N} \ell \varphi s_{\ell}^z. \tag{8}$$

This transformation rotates the $\ell$th spin around the $z$ axis by angle $\ell\varphi$ and shifts the spectrum (7) by the wave number $\varphi$.

This shift is analogous to the phase shift experienced by electrons on a ring threaded by a constant magnetic flux ($\Phi \sim N\varphi$) [12].

We also note that the ferromagnetic and the antiferromagnetic cases are equivalent in the sense that the canonical transformation (8) with $\varphi = \pi$ transforms them into each other. For a finite periodic chain, $H^M$ is transformed to $H^{XY}$, but with a twisted boundary condition ($\varphi \neq \pi$).

As the transformation (8) does not change the $z$ component of the spins, we find that

$$\langle s_{\ell}^z \rangle = \frac{1}{\pi}\arcsin\frac{h}{\sqrt{1+\lambda^2}} \tag{9}$$

and the correlation function

$$\rho^z(r) = \langle s_{\ell}^z s_{\ell+r}^z \rangle = -\frac{1}{\pi^2 r^2}\sin^2\left[ r\arccos\frac{h}{\sqrt{1+\lambda^2}} \right] \tag{10}$$

have their equilibrium form but at a different field $\tilde{h} = h/\sqrt{1+\lambda^2}$ (angular brackets denote the expectation value in the ground state of $H^M$). One can see that, for $h \geq \sqrt{1+\lambda^2}$ ($\tilde{h} \geq 1$), the spins are parallel to the field and there is no current in the system. For $h < \sqrt{1+\lambda^2}$ ($\tilde{h} < 1$) the current has a simple form

$$j^M = \langle J^M/N \rangle = \frac{\lambda\sqrt{1+\lambda^2-h^2}}{\pi(1+\lambda^2)}, \tag{11}$$

and one can observe that the maximum current, reached in the limit of $\lambda \to \infty$, is given by $1/\pi$.

The introduction of the transformation (8) simplifies the calculation of the correlations $\rho^x(r) = \langle s_{\ell}^x s_{\ell+r}^x \rangle = \rho^y(r)$ in the ground state of $H^M$:

$$\rho^x(r) = \cos(r\varphi)\langle Q\psi_0|s_{\ell}^x s_{\ell+r}^x|Q\psi_0 \rangle$$
$$+ \sin(r\varphi)\langle Q\psi_0|s_{\ell}^x s_{\ell+r}^y|Q\psi_0 \rangle, \tag{12}$$

where $\psi_0$ is the ground state of $H^M$, while $Q\psi_0$ is that of $H^{XY}$ at a field $\tilde{h}$. Without any current, we have $\langle s_{\ell}^x s_{\ell+r}^y \rangle = 0$ and, furthermore, the $r \to \infty$ behavior of the $\rho^x(r)$ correlation function [13] is given by

$$\rho^x(r; j^M = 0) \approx (1 - h^2)^{1/4}\frac{C}{\sqrt{r}}, \tag{13}$$

where $C = e^{1/2}2^{-4/3}A^{-6} \approx 0.147$ ($A \approx 1.282$ is the Glaisher constant). Using (12), one can obtain then the following simple form in the $r \to \infty$ limit:

$$\rho^x(r) \approx (1 - \tilde{h}^2)^{1/4}\frac{C}{\sqrt{r}}\cos(r\varphi). \tag{14}$$

Thus we find that the correlations decay by a power law and show oscillatory behavior in the current-carrying states. This is similar to what has been observed in the transverse Ising model, but there are some differences. In the Ising case, an exponential decay of correlations changes into a power-law

form as the current is switched on. In the *XY* case, on the other hand, one has power-law correlations already in the equilibrium state. The magnetization current does not change the power law and leaves its exponent intact as well. The increase of correlations appears as the increase in the amplitude of the power law (note that $\tilde{h} = h\cos\varphi \leq h$).

## III. ENERGY CURRENT

Since the energy is a conserved quantity as well, one can investigate the effects of the energy current. The local energy (the contribution of the $\ell$th spin to the total energy) satisfies a continuity equation with the local energy current $j_\ell^E$ and its sum over $\ell$ (the total energy current) has the form

$$J^E = \sum_{\ell=1}^{N} [s_\ell^z(s_{\ell-1}^y s_{\ell+1}^x - s_{\ell-1}^x s_{\ell+1}^y) + h(s_\ell^x s_{\ell+1}^y$$

$$- s_\ell^y s_{\ell+1}^x)]. \quad (15)$$

One can easily show that $[H^{XY}, J^E] = 0$ and diagonalizing the Hamiltonian

$$H^E = H^{XY} - \lambda J^E, \quad (16)$$

one obtains the lowest-energy eigenstates of $H^{XY}$ in the presence of a given $J^E$.

Using the standard transformations to fermions again, the spectrum is obtained as

$$\Lambda_k = (-\cos k - h)(1 - \lambda \sin k) \quad (17)$$

and the modes with negative energy are occupied in the ground state of $H^E$. Although the $k \rightarrow -k$ symmetry of the spectrum is broken for $\lambda \neq 0$, the ground state remains that of $H^{XY}$ for $\lambda \leq 1$ and, accordingly, no energy current flows through the system. This rigidity of the ground state against $\lambda$ is a consequence of the fact that the fermionic spectrum of $H^E$ has a product form (17) and the second factor is positive for $\lambda < 1$. The first and second factors in $\Lambda_k$ change sign at $\pm(\pi/2 + k_h)$ (for $h \leq 1$) and $\pi/2 \pm k_\lambda$ (for $\lambda \geq 1$), respectively. The "critical momenta" $k_h$ and $k_\lambda$ are defined here such that they take values $0 \leq k_h \leq \pi/2$ and $0 \leq k_\lambda \leq \pi/2$ and one has $k_h = \arcsin(h)$ and $k_\lambda = \arccos(\lambda^{-1})$.

One can study the ground state as a function of $h$ and $\lambda$, but we are more interested in the physical quantities as functions of $h$ and $j^E = \langle J^E/N \rangle$. Thus first we calculate $j^E$,

$$j^E = \begin{cases} \dfrac{1}{2\pi}(1 + h^2 - \lambda^{-2}) & \text{for } k_h \leq k_\lambda \\[2ex] \dfrac{h}{\pi}\sqrt{1 - \lambda^{-2}} & \text{for } k_h \geq k_\lambda \text{ or } h, \lambda \geq 1 \\[2ex] 0 & \text{for } \lambda \leq 1, \end{cases} \quad (18)$$

and then express all the $\lambda$ dependences in terms of $j^E$. We can then obtain an $h - j^E$ phase diagram as shown in Fig. 1, where the phases, discussed below in more detail, are distinguished by symmetries of the regions of occupied states in the $k$ space.
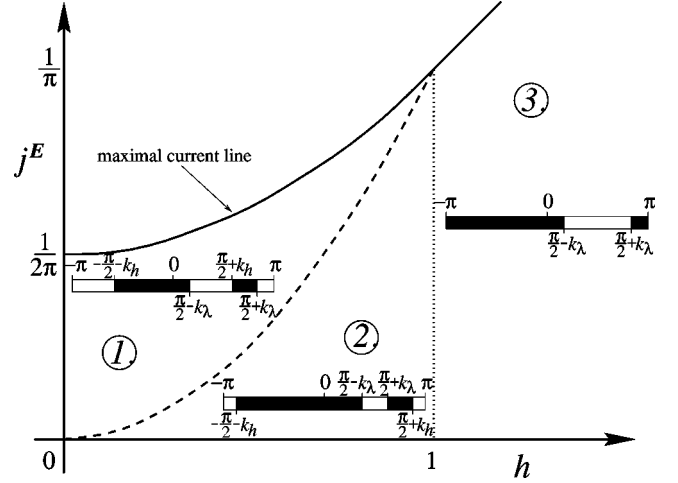


FIG. 1. Phase diagram of the ground state of the transverse *XY* model in the presence of energy current. The black parts of the rectangles denote the wave numbers of the occupied fermionic modes ($-\pi \leq k < \pi$). The dashed line is a high-symmetry transition line between regions of 1 ($M^z = 0$) and 2 ($M^z \neq 0$). There are no states above the maximal current line and region 3 can be mapped onto the vertical dotted line ($h = 1$).

### A. Phase diagram

Let us begin the analysis of the phase diagram by first describing it in terms of the behavior of currents and of the transverse magnetization. As shown in Fig. 1, there is a maximal current for every value of $h$

$$j_{max}^E = \begin{cases} (1 + h^2)/2\pi & \text{for } h \leq 1 \\ h/\pi & \text{for } h \geq 1 \end{cases} \quad (19)$$

and no state exists above $j_{max}^E$.

We can divide the $h$-$j^E$ phase diagram into three regions. The only interesting areas are 1 and 2 and their boundaries. In region 3, the ground state is the same along the $j^E = \text{const} \times h$ lines ($h \geq 1$) thus the $h = 1$ line contains all the information about this region. Below, we restrict the discussion to the $h \leq 1$ part of the phase diagram with the understanding that the $h = 1$ line represents region 3.

As can be seen from Eq. (15), the energy current has two parts: the term containing $h$ is proportional to the magnetization current $(-hJ^M)$, while the other term is the current of the interaction energy. The distinguishing features of regions 2 and 3 are that the current of interaction energy is zero [14], while the transverse magnetization $M^z$ is nonzero in the ground state.

For any fixed value of $h$, the magnetization decreases with increasing $j^E$ and $M^z$ becomes zero on the line $k_h = k_\lambda$ corresponding to $j^E = h^2/\pi$. On this line, the magnetization-current part of $j^E$ saturates and, upon increasing $j^E$, we enter region 1 where the interaction part of the current starts to flow. Another characteristic feature of region 1 is that $M^z = 0$ throughout this region.

One tends to conclude at this point that the line separating regions 1 and 2 is a line of second order, nonequilibrium phase transitions with $M^z$ being the order parameter. This notion is also supported by the facts that several quantities such as $\rho^z(1)$ and $\rho^x(1)$ have a jump in their first derivatives

when crossing the transition line and, furthermore, that the correlations are enhanced (change from $r^{-1}$ to $r^{-1/2}$) on this line (see Sec. III B). If this was a phase transition, however, it was certainly a transition not in the usual sense. The symmetry of the ground state is the same on both sides of the transition line and the $M^z=0$ result in region 1 is not a consequence of the up-down symmetry of the ground state. The magnetization is zero in 1 because the motion of the zeros of the dispersion relation (17) conspire to keep the ground state at half filling. We emphasize, however, that the half filling does not mean that the ground state has a symmetry with respect to global spin flip $s_i^z \rightarrow -s_i^z$.

It is interesting to note that the symmetry of the ground state is higher on the transition line than on either side of it. Indeed, on this ''high''-symmetry line, the ground state is symmetric with respect to rotation of the spins around the $x$ axis by $\pi$, followed by a spatial reflection mapping site $i$ to $L+1-i$. The Hamiltonian $H^{XY}$ has no such symmetry and, off the transition line, the ground state does not have such symmetry either. Thus we can see here an example where the increase of current in a system leads, at a particular value of the current, to symmetry enhancement. The reason for the increase of symmetry is obviously some level crossing coming from the interplay of the current operator and the original Hamiltonian. One might speculate that the occurrence of such symmetry enhancements is not an accidental but a general feature of current-carrying systems.

### B. Correlations

The $\rho^z(r)$ correlations can be calculated easily and, just as in the equilibrium case, one finds $\rho^z(r) \sim r^{-2}$. The difference from the equilibrium is that the oscillatory modulation of the $r^{-2}$ decay (present in equilibrium for $h \neq 0$) becomes more complex. Such modulation has been observed in case of the imposed magnetic current (see Sec. II) as well as in the transverse Ising model with energy current [6]. The exponent of the power-law decay, however, is unchanged when the currents are introduced in all of the above examples. Thus it seems that $\rho^z(r)$ correlations are not too sensitive to the presence of currents. A possible reason for this apparent rigidity is, perhaps, the lack of internal interactions among the $z$ components of the spins.

It is harder to calculate the $\rho^x(r)=\rho^y(r)$ correlations but they show a more interesting behavior. Some of our results described below are exact and were derived by combining the Wick theorem and the spin rotation transformation (8), which relates the correlation functions between states where the ground-state occupation pattern in $k$ space is identical up to shifts $k \rightarrow (k+\alpha) \mod 2\pi$. As we shall see, these exact results are restricted to the boundaries of regions 1 and 2. At a general point $(h,j^E)$, we were able to calculate $\rho^x(r) \equiv \rho^x(r;h,j^E)$ numerically (for $r \lesssim 100$ lattice spacings) using the fact that the square of the correlation can be expressed as a determinant of a $2r \times 2r$ matrix with exactly calculable elements.

Let us start by enumerating the exact results. The boundaries of region 2 are discussed in points (i)–(iii), while the boundaries of 1 are treated in points (iii)–(iv).

(i) As discussed in Sec. II, the correlations in the transverse *XY* model without current, i.e., on the line ($0<h$

$<1,j^E=0$), are known [13] and the $r \rightarrow \infty$ asymptotics of $\rho^x$ is given by

$$\rho^x(r;h,j^E=0) \sim C(1-h^2)^{1/4} r^{-1/2}. \qquad (20)$$

(ii) The correlations on the $h=1$ line can be related to the equilibrium case ($0<h<1,j^E=0$) and one finds

$$\rho^x(r;1,j^E) = \rho^x(r;\sqrt{1-\pi^2 j^{E^2}},0)\cos\left(\frac{\pi}{2}r\right). \qquad (21)$$

Thus the large-distance behavior is given by

$$\rho^x(r;1,j^E) = C\sqrt{\pi j^E} r^{-1/2}\cos\left(\frac{\pi}{2}r\right). \qquad (22)$$

Since the whole region 3 phase can be projected onto the $h=1$ line, we find that correlations decay as $r^{-1/2}$ for $h \gtrsim 1$.

(iii) The correlations on the high-symmetry line (i.e., on the boundary between 1 and 2) can also be related to equilibrium:

$$\rho^x(r;h,j^E=h^2/\pi) = \rho^x(r;0,0)\cos(k_h r)$$
$$\sim Cr^{-1/2}\cos(k_h r), \qquad (23)$$

so we find again an $r^{-1/2}$ decay in the $r \rightarrow \infty$ limit.

(iv) On the line of maximal current $[h \leq 1, j_{max}^E = (1+h^2)/2\pi]$, the correlations can be expressed in terms of those on the line ($h=0,j^E$):

$$\rho^x(r;h,j_{max}^E) = \rho^x\left(r;0,\frac{1-h^2}{2\pi}\right)\cos\left(\frac{\pi}{2}r\right). \qquad (24)$$

Unfortunately, this does not help in calculating the $r \rightarrow \infty$ behavior.

(v) The long-range behavior of the correlations at the intersection point $[0,1/2\pi]$ of the ($h=0,j^E$) and the ($h,j_{max}^E$) lines is also calculable:

$$\rho^x\left(r;0,\frac{1}{2\pi}\right) = \begin{cases} 4\left[\rho^x\left(\frac{r}{2};0,0\right)\right]^2 \sim \frac{1}{r} & \text{for } \frac{r}{4} \text{ integer} \\ 0 & \text{otherwise} \end{cases} \qquad (25)$$

and it is remarkable that the correlation function in this point decays as $1/r$ instead of $1/\sqrt{r}$.

The exact results can be summarized as follows. The $\rho^x=\rho^y$ correlations decay as $1/\sqrt{r}$ on the boundaries of region 2, while a $1/r$ decay can be observed in the upper-left corner $[0,1/2\pi]$ of the phase diagram.

Numerical calculations suggest, however, that the $1/r$ asymptotics is more general than it looks from the exact results. There is a strong indication that the large-distance asymptotics is actually $1/r$ everywhere in region 1 and 2 apart from the boundaries of region 2. Figure 2 shows an example of numerical results at a general point of the phase diagram. The following formula gives an excellent fit to the numerical data throughout the phase diagram (except very close to the lines with the $1/\sqrt{r}$ behavior):
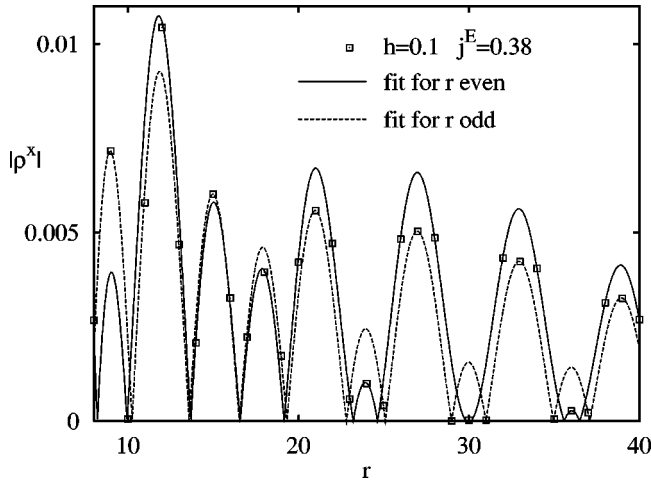
FIG. 2. Absolute value of the $\rho^x(r) = \langle s^x_\ell s^x_{\ell+r} \rangle$ correlation function at a general point of the phase diagram. The distance $r$ is measured in units of lattice spacing. The long-range decay of $\rho^x(r)$ is fitted by expression (26) (solid and dashed lines) (for this point of the phase diagram one has $a_1 \approx a_2 \approx 0.0934$ and $a_3 \approx 0.0469$).

$$\rho^x(r) = \begin{cases} [a_1 \cos(k_h r) + a_2 \cos(k_\lambda r)]/r & \text{for } r \text{ even} \\ [a_3 \cos(k_h r) + a_2 \cos(k_\lambda r)]/r & \text{for } r \text{ odd}. \end{cases} \tag{26}$$

The $a_i$ coefficients are functions of $h$ and $j^E$, and for the 1 phase $a_1 = a_2$ seems to be valid.

As one can see from Eq. (26), the amplitude of the $1/r$ decay is modulated with the critical wave numbers $k_h$ and $k_\lambda$. On the high-symmetry transition line we have $k_h = k_\lambda$ and the transition across this line takes us from region 1 where $k_h < k_\lambda$ to region 2 where $k_h > k_\lambda$. Thus one can view the high-symmetry line as a line of degeneracy where two characteristic wavelengths of the system become equal.

This transition may resemble transitions arising from competing wavelengths but, actually, here we do not have a competition between $k_h$ and $k_\lambda$. Due to the product form of $\Lambda_k$, $k_h$ is independent of $\lambda$ while $k_\lambda$ is independent of $h$. Nevertheless, this transition does have similarities to second-order transitions in that the correlations decay more slowly ($1/r \to 1/\sqrt{r}$) and, furthermore, one can observe scaling upon approach of the transition line. In order to see this, let us assume that the distance from the transition line $k_h - k_\lambda$ provides the single diverging length scale that generates the $1/r \to 1/\sqrt{r}$ crossover in correlations. Then one should observe scaling when plotting the ratio

$$\frac{\rho^x(r; h, j^E)}{\rho^x(r; h_c, j^E_c)} = \Phi\left(\frac{|k_h - k_\lambda|}{2} r\right), \tag{27}$$

where $(h_c, j^E_c)$ is a point on the transition line and the $h \to h_c, j^E \to j^E_c$ limit is taken. Note that the long-range behavior of the denominator ($\rho^x$ on the phase transition line) is known [Eq. (23)]. As one can see from Fig. 3, the data collapse is excellent, thus supporting the assumption of scaling (27).

It is interesting to note that the scaling function appears to be the same on the both sides of the phase-transition line.
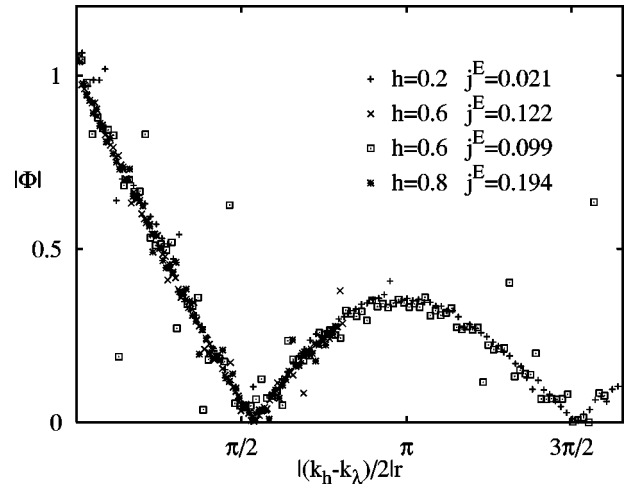


FIG. 3. Absolute value of the scaled $\rho^x$ correlation function at four points of the phase diagram near the boundary between regions 1 and 2. Two points are above and two are below this high-symmetry line. The data points showing large deviations from scaling come from arguments where both the numerator and the denominator in Eq. (27) are close to zero.

Furthermore, $\Phi$ is independent of the crossing point $(h_c, j^E_c)$ unless we are close to the zero-field equilibrium point ($h = 0, j^E = 0$). In this sense we have the kind of universality usually observed in critical phase transitions.

The equilibrium point ($h = 0, j^E = 0$) is the end point of the high-symmetry line. At this point, the ground-state symmetry is higher than on the line and so we may expect that, provided scaling is still present, the scaling function would be different. This is indeed what we observe. Approaching the point ($h = 0, j^E = 0$) along the ($h = 0, j^E \to 0$) line, one has again a diverging length scale proportional to $1/k_\lambda$ and one can search again for scaling in the correlation function

$$\frac{\rho^x(r; 0, k_\lambda)}{\rho^x(r; 0, 0)} = \Psi(k_\lambda r). \tag{28}$$

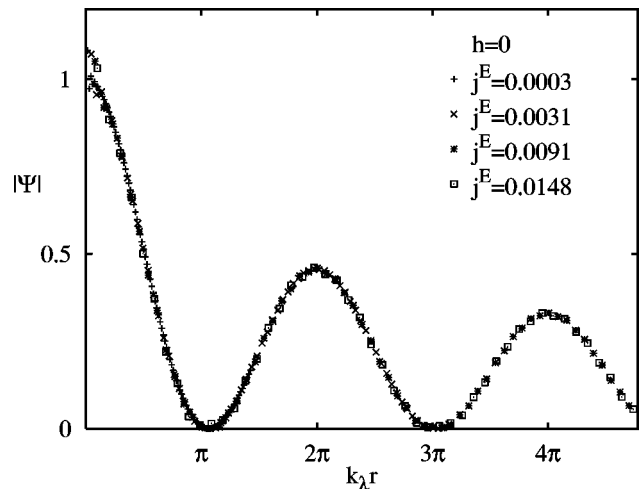As shown in Fig. 4, scaling is indeed seen and the scaling



FIG. 4. Absolute value of the scaled $\rho^x$ correlation functions on the $h = 0$ line near $j^E = 0$.

function is significantly different from that found away from the $(h=0, j^E=0)$ point.

The numerical results presented above (as well as other data gathered in our explorations of the phase diagram) suggest strongly that $\rho^x \propto 1/r$ for generic current-carrying states. Slower decay $\rho^x \propto 1/\sqrt{r}$ is observed only on the boundaries of region 2 and the crossover between the $1/r$ and $1/\sqrt{r}$ behaviors can be understood in terms of single-length scale scaling. It is intriguing that there is a simple correspondence between the types of decay of correlations and the ''band structure'' of the ground state. The lines of slower decay of correlations coincide with those lines where the ground state is built by a single band of excitations in momentum space, whereas in all regions of $1/r$ decay, the filling pattern of the ground state splits into two separate bands (Fig. 1).

Regarding the interplay of currents and correlations, these results leave us with the following conclusions. First, we find that the large-distance correlations are not necessarily increased by switching on a current. Second, it is found that the equilibrium power-law correlations are not destroyed by the current; only the exponent in the power law is increased. This strengthens previous observations that currents and power-law correlations are intimately related. Third, we find that the increase of current may lead to interesting phase-transition-like behavior related to the *increase of symmetry* at special values of the current.

## IV. FINAL REMARKS

A general conclusion we can draw from the present study of the transverse *XY* model and from a comparison with the results on the transverse Ising model [6] is that currents appear to generate and maintain power-law correlations. An interesting feature of the *XY* model that may also have some generality is the increase of the symmetry of the ground state at special values of the energy current. This feature should certainly be searched for in other models as well as in experiments. It should be recognized, however, that both the *XY* and the transverse Ising models are integrable and, consequently, they are special in that conductivity and, in particular, the thermal conductivity are ideal for them (not only at zero but also at nonzero temperatures) [15–17]. Thus it is an important next step to find out whether nonintegrable models have the same connection between currents and power-law correlations and, furthermore, whether they show any additional general features.

[1] A simple experimental system displaying this behavior is a liquid that develops long-range spatial correlations if driven out of equilibrium by an imposed temperature gradient (heat current); R. Schmitz, Phys. Rep. **171**, 1 (1988).

[2] For a recent review see B. Schmittmann and R. K. P. Zia, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and J. L. Lebowitz (Academic, New York, 1995), Vol. 17.

[3] Remarkably, even local currents can yield the same effect. B. Schmittmann, Europhys. Lett. **24**, 109 (1993); K. E. Bassler and Z. Rácz, Phys. Rev. Lett. **73**, 1320 (1994); Phys. Rev. E **52**, R9 (1995).

[4] A simple example is the Bénard instability [M. C. Cross and P. C. Hohenberg, Rev. Mod. Phys. **65**, 851 (1994)], where the stationary convection cells become time dependent as the current of heat is increased, and, above a critical current, the time evolution becomes chaotic.

[5] Reviews of one-dimensional models can be found in *Nonequilibrium Statistical Mechanics in One Dimension*, edited by V. Privman (Cambridge University Press, Cambridge, 1997).

[6] T. Antal, Z. Rácz, and L. Sasvári, Phys. Rev. Lett. **78**, 167 (1997).

[7] E. Lieb, T. Schultz, and D. Mattis, Ann. Phys. (N.Y.) **16**, 407 (1961).

[8] I. E. Dzyaloshinskii, J. Phys. Chem. Solids **4**, 241 (1958); T. Moriya, Phys. Rev. **120**, 91 (1960).

[9] F. C. Alcaraz, M. N. Barber, and M. T. Batchelor, Ann. Phys. (N.Y.) **182**, 280 (1988).

[10] A. A. Zvyagin, J. Phys.: Condens. Matter **3**, 3865 (1991); O. V. Derzhko and A. Ph. Moina, Condens. Matter Phys. **3**, 3 (1994).

[11] G. Schütz, Phys. Rev. E **49**, 2461 (1994).

[12] N. Byers and C. N. Yang, Phys. Rev. Lett. **7**, 46 (1961).

[13] E. Baruch and B. McCoy, Phys. Rev. A **3**, 786 (1971); B. M. McCoy and T. T. Wu, *The Two-Dimensional Ising Model* (Harvard University Press, Cambridge, MA, 1973).

[14] Note that although only magnetization current flows in these states, they nevertheless differ from the current-carrying states studied in Sec. II. There the expectation value of the interaction part of the current takes a nonzero value.

[15] B. S. Shastry and B. Sutherland, Phys. Rev. Lett. **65**, 243 (1990); **65**, 1833 (1990).

[16] H. Castella, X. Zotos, and P. Prelovsek, Phys. Rev. Lett. **74**, 972 (1995); X. Zotos, F. Naef, and P. Prelovsek, Phys. Rev. B **55**, 11 029 (1997).

[17] K. Saito, S. Takesue, and S. Miyashita, Phys. Rev. E **54**, 2404 (1996).